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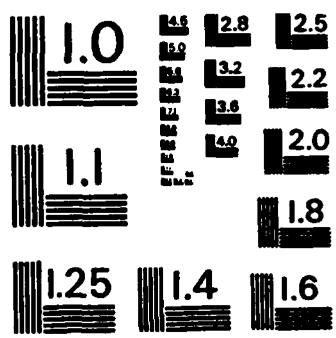

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NECESSARY AND SUFFICIENT CONDITIONS FOR SINGLE-PEAKEDNESS ALONG A LINEARLY ORDERED SET OF POLICY ALTERNATIVES

by

PETER J. COUGHLIN AND MELVIN J. HINICH

TECHNICAL REPORT NO. 376
April 1982

A REPORT OF THE
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY
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THE ECONOMICS SERIES

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NECESSARY AND SUFFICIENT CONDITIONS FOR SINGLE-PEAKEDNESS ALONG A LINEARLY ORDERED SET OF POLICY ALTERNATIVES*

by

Peter J. Coughlin** and Melvin J. Hinich***

1. Introduction

Median voter results have played an important role in the development of social choice theory and in economic analyses of the public sector. The various discussions of these results which have appeared in the literature can be divided into two basic categories: those in which voter preferences are assumed to be single-peaked over an ideological continuum and those in which voters' preferences are assumed to be single-peaked over a unidimensional set of economic policies (or social alternatives). The earliest example of the first category is Hotelling [1929]. This approach also provided the basis for many of the arguments put forth in Downs [1957]. More recently, this version of the median voter result has appeared in Mueller [1976, 1979], McKenzie and Tullock [1978], Brams [1978], Feldman [1980], and elsewhere. Early examples of the second category include Black [1948]

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and Arrow [1951]. Since the appearance of these seminal works, analyses of majority rule with multidimensional sets of economic policies (or social alternatives) have also been developed.[1]

This paper is concerned with the question: Given a world in which both ideological labels and economic policies affect voter decisions, when is the basic assumption of Hotelling, Downs, et al. of single-peakedness along an ideological continuum justified? In particular, ~~we study~~ this question is studied for situations in which the link between ideological labels and economic policies is the one suggested by Downs [1957] in his explanation of why he focused entirely on the first of these.[2] In this paper, this link is formalized by the assumption that each voter has a "predictive map" which assigns to each ideological label (or alternative predictive element) the economic policies which he thinks can be expected if a politician (or some other social alternative) identified with that label (or element) is the social choice.

This analysis follows the earlier work on this problem by Hinich and Mackay [1980] and Hinich and Pollard [1981]. Using this same link between predictive sets (such as ideological continua) and economic policies, these recent papers have established that if (i) the predictive set is a continuum on the real line, (ii) there is an m-dimensional policy space $(1 \leq m < \infty)$, on which voters have ellipsoidal preferences,[3] and (iii) the predictive maps for the voters are affine functions, then the indirect voters' preferences which are defined on the predictive dimension are single-peaked (with respect to the natural linear order

for this dimension). As a consequence of its conclusion, certain well-known theorems can be applied in these circumstances to obtain further results. In particular, it now follows that there is a location on the predictive dimension such that any candidate identified with this location will receive at least as many votes as any other candidate.[4/] Additionally, any such position is also a median for the distribution of voters' ideal points (for the indirect preferences) on this dimension. This has provided new equilibrium existence and location results for electoral competitions in societies with multidimensional economic policy spaces.

The analysis in this paper extends this earlier work by: determining the predictive maps for which the results described in the previous paragraph generalize when voter preferences are ones which are naturally inherited from private sector preferences. For a careful analysis of the properties of such preferences which are inherited in this way — and those which are not — we refer the reader to Denzau and Parks [1977, 1979]. Using their work as a point of departure, we analyze societies whose voter preferences are reflexive, connected, transitive, regular, weakly convex and convex.[5/] Individuals whose public-sector preferences satisfy these assumptions will be referred to (in the text) as "economic decision makers."

In our analysis we also relax the assumption that the predictive set is a continuum to the assumption that it is a linearly ordered set. We additionally relax the initial assumptions on the voters' predictive maps by studying the class of continuous predictive maps, rather than

starting out with linearity. The one place where a strengthening occurs in our analysis is in our assumption that the set of predictive elements is compact. This, however, is quite natural for such sets.

2. Societies with Varying Voter Predictions

Let $X \subset E^m$ denote a convex set of possible economic policies (contained in an m-dimensional Euclidean space). All topological statements concerned with this space will be with respect to the usual (relative) Euclidean topology. We use a Euclidean policy space because this is the type of social alternative set which is of interest in most of the literature on the democratic selection of economic policies (see the references cited in footnote 1). Furthermore, thanks to the work of Denzau and Parks [1977, 1979], this is a context in which the nature of voters' preferences on public sector alternatives is known.

The society's voters are indexed by the elements, i, in the set $N = \{1,2,\ldots,n\}$, where n is finite. The preferences of any individual (or voter) i on the set X will be specified by the subset $R_i \subset X \times X$. As stated in the introduction, we assume that each R_i is reflexive, connected, transitive, regular, weakly convex and convex.[5/] $P(R_i)$ is used for the asymmetric part[6/] of R_i, and $I(R_i)$ is used for the symmetric part[7/] of R_i. The n-tuple $(R_1,\ldots,R_n)$ of individual preferences in a particular society is that society's "profile on X." The collection of all possible profiles which satisfy the above assumptions (for a particular n) is denoted by $R(n)$.

Let Π denote a set of "predictive elements" for the voters (such as one of the predictive sets described in the introduction). We will assume that Π has at least three elements. The symbol $\leq_o$ will denote a natural linear order[8/] on Π. The asymmetric part of $\leq_o$, in particular, will be written as $<_o$. The existence of such an ordering means, of course, that Π is essentially one-dimensional in nature (e.g., see the closely related discussion of strong ordering in Chapter 7 of Arrow [1963]). The reason why this is the most appropriate setting for our analysis is the fact that, as a consequence of the results of Kramer [1976], Wagstaff [1976], and others, we already know a priori that any conditions which could assure single-peakedness on multidimensional predictive sets would, necessarily, be highly restrictive.

All topological statements about Π are made with respect to the order topology[9/] on this set which corresponds to the natural linear order $\leq_o$ (as in Denzau and Parks [1975]). Assume that Π is compact.

We also assume throughout that all of the voters agree on the location in Π which corresponds to any particular candidate, but may disagree about the economic policies which will follow if the candidate is elected. We formulate this by assuming that each voter has a "predictive map," $w_i(\pi)$, which is a continuous[10/] function from Π into X. This, in turn, implies that "indirect preferences on Π" are defined for each $i \in N$ by the relation

(2.1) $$(\pi_1,\pi_2) \in Q_i \text{ if and only if } (w_i(\pi_1), w_i(\pi_2)) \in R_i$$

for any $\pi_1, \pi_2 \in \Pi$. We use $P(Q_i)$ for the asymmetric part of Q_i, and $I(Q_i)$ for the symmetric part of Q_i. The n-tuple $(Q_1,\ldots,Q_n)$ which occurs in a particular society is its "indirect profile" or "profile on Π."

An indirect profile on Π is said to be "single-peaked"[11/] with respect to the linear order $\leq_o$ if and only if, for each $i \in N$, there are unique $a_i, b_i \in \Pi$ with $a_i \leq_o b_i$ such that

(2.2) a) $\pi_1 <_o \pi_2 \leq_o a_i$ implies $(\pi_2,\pi_1) \in P(Q_i)$

b) $a_i \leq_o \pi_1 \leq_o \pi_2 \leq_o b_i$ implies $(\pi_2,\pi_1) \in I(Q_i)$

c) $b_i \leq_o \pi_2 <_o \pi_1$ implies $(\pi_2,\pi_1) \in P(Q_i)$, and

d) $\pi_1 <_o a_i$ or $b_i <_o \pi_1$, and $a_i \leq_o \pi_2 \leq_o b_i$

implies $(\pi_2,\pi_1) \in P(Q_i)$

For a standard representation of preferences which satisfy these requirements see Figure 1.

We will conclude this section by stating (and then discussing) a theorem which specifies restrictions on the predictive maps of the voters which are both necessary and sufficient to be able to conclude that every indirect profile on Π is single-peaked with respect to the linear order $<_o$. In order to be able to state this theorem precisely, we will have to review a few more definitions.

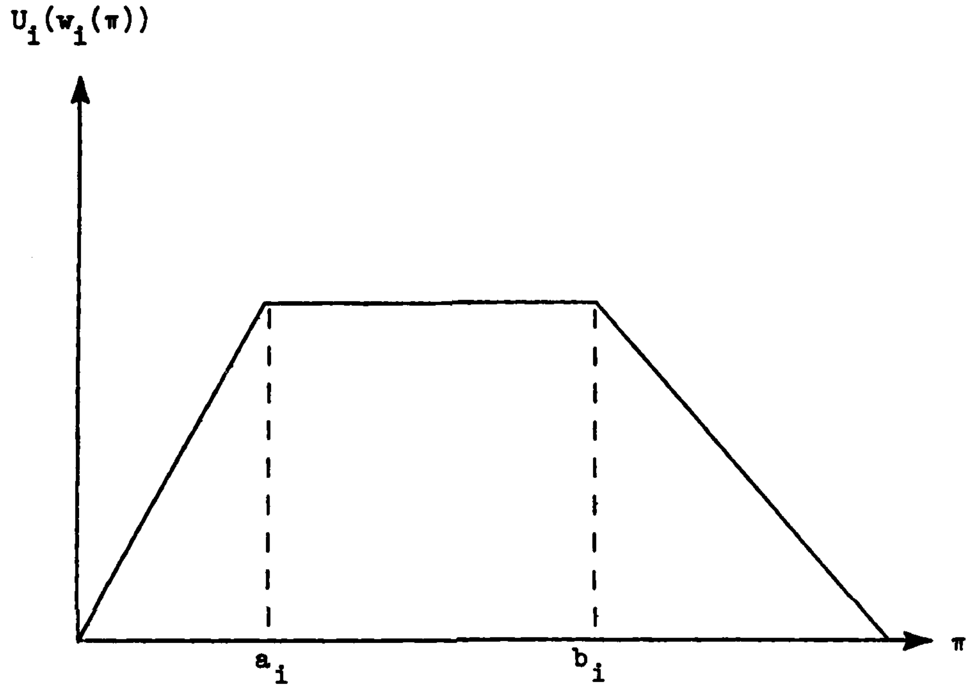


Figure 1

First of all recall that, for a given pair of elements x,y in a convex set $X \subset E^m$, the set of points in X, $I(x,y)$, such that $\alpha \cdot x + (1 - \alpha) \cdot y$ for $0 \leq \alpha \leq 1$ is called "<u>the</u> linear interval connecting the points x and y." Therefore, any set of points which is the linear interval connecting the points x and y for some pair $x, y \in X$ is called "<u>a</u> linear interval in X." Any linear interval $I(x,y)$ is, of course, naturally ordered by α, i.e., by the relation $\geq_\alpha$ which is specified by

$$(2.3) \qquad [z_1 = \alpha_1 x + (1 - \alpha_1)y \geq_\alpha z_2 = \alpha_2 x + (1 - \alpha_2)y] \iff \alpha_1 \geq \alpha_2$$

for $\alpha_1, \alpha_2 \in [0,1]$. Furthermore, any linear interval $I(x,y) = I(y,x)$ has two such orderings. Therefore, a predictive mpa, $w_i(\pi)$, whose range is contained in a linear interval, $I(x,y)$, is said to be "strictly monotonic on this interval" if and only if

$$(2.4a) \qquad \pi_1 <_\circ \pi_2 \Rightarrow w_i(\pi_1) <_\alpha w_i(\pi_2) \quad , \quad \forall \ \pi_1, \pi_2 \in \Pi \quad , \quad \underline{\text{or}}$$

$$(2.4b) \qquad \pi_1 <_\circ \pi_2 \Rightarrow w_i(\pi_1) >_\alpha w_i(\pi_2) \quad , \quad \forall \ \pi_1, \pi_2 \in \Pi \quad .$$

It should be observed that these range restrictions (which appear as condition 2) in the theorem below) are satisfied by a predictive map, $w_i(\pi)$, if and only if there exist vectors c_i, $d_i \in X$ and a strictly monotonic (real-valued) function, $f_i(\pi)$, such that

$$(2.5) \qquad w_i(\pi) = c_i + f_i(\pi) \cdot d_i \quad , \quad \forall \ \pi \in \Pi \quad .$$

Finally, we will also use the notation $\overline{W}(n)$ to denote a subset of the class of all possible n-tuples of predictive maps, $(w_1(\pi),\ldots,w_n(\pi))$.

Using these definitions, we can now state our first theorem:

<u>Theorem 1</u>: Every pair in $R(n) \times \overline{W}(n)$ has an indirect profile on Π which is single-peaked with respect to $\leq_o$ if and only if each n-tuple in $\overline{W}(n)$ is such that each voter's predictive map either

1) is constant, or

2) has its range contained in a linear interval[12/] and is strictly monotonic on this interval.

Examples of predictive maps which satisfy 2) are given by the linear and affine predictive maps which have been studied in Hinich and Mackay [1979] and Hinich and Pollard [1981]. Therefore, this theorem establishes that the basic results on single-peakedness in these papers are ones which extend for all societies with economic decision makers. However, the restrictions on the voters' predictive maps specified by this theorem are strictly weaker than the restrictions on voters' predictive maps which were studied in these papers. Hence, it also reveals that certain piecewise linear functions (e.g., see Figure 2) and other nonlinear functions (e.g., see Figure 3) are also sufficient for the preferences of economic decision makers to always lead to indirect preferences on Π which are single-peaked with respect to $\leq_o$.

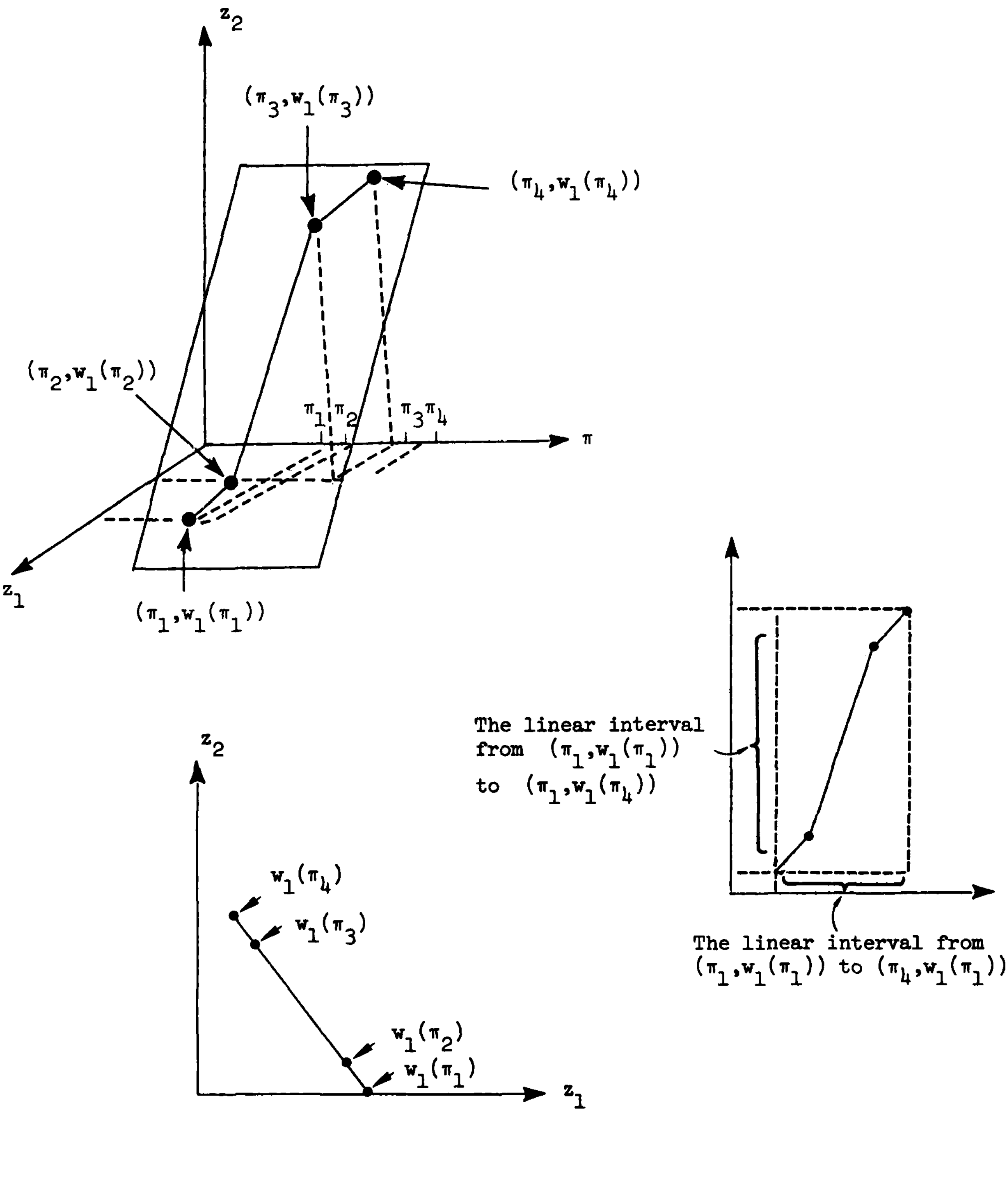


Figure 2: Three views of a map, $w_1:\pi\to Z_1\subset E^2$

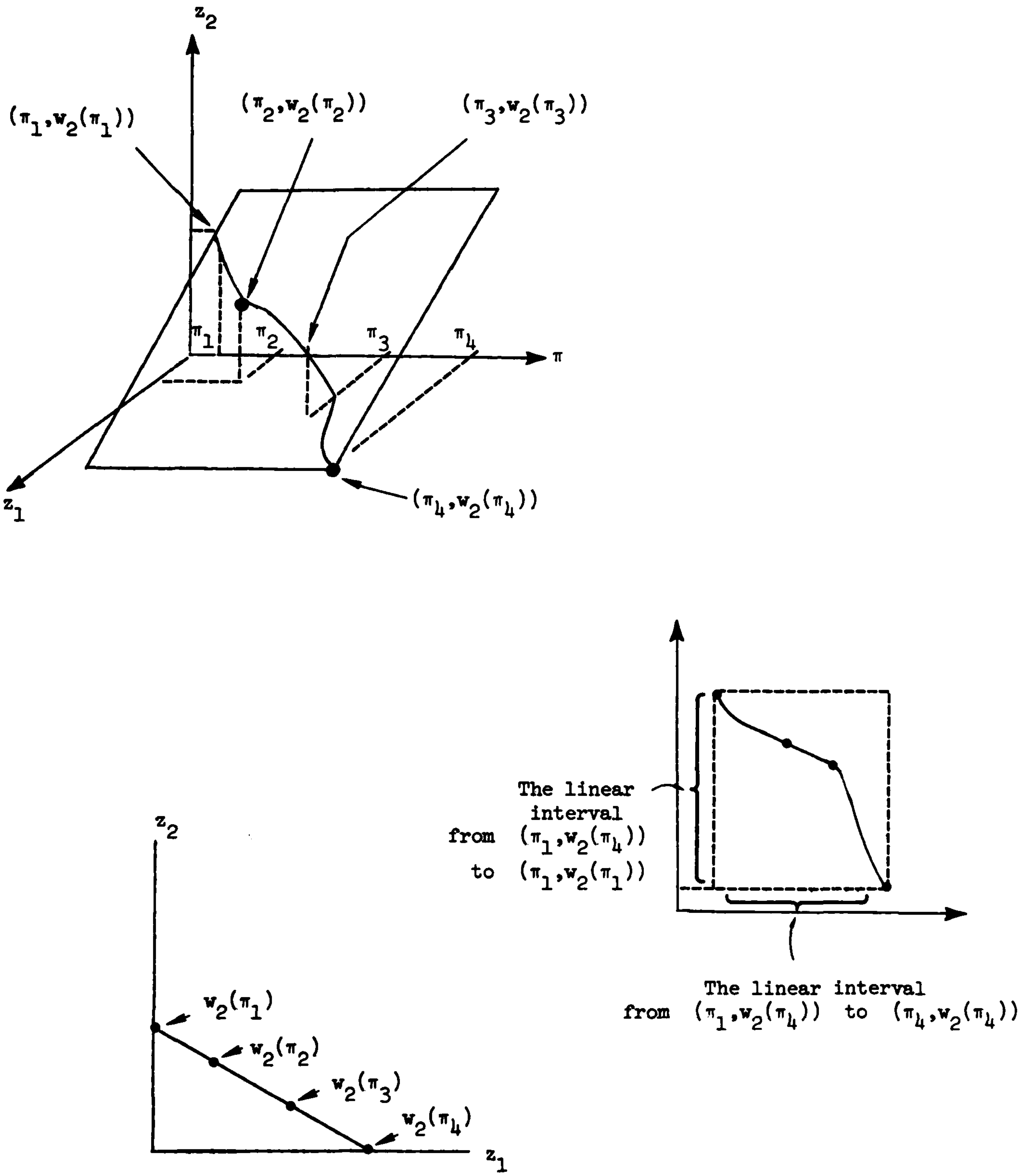


Figure 3: Three views of a map, $w_2 : \pi \to Z_2 \subset E^2$

Furthermore, by its necessity statement, it establishes that the condition which have been specified here are ones which are minimally sufficient[13/] for societies with economic decision makers.

When the society's policy space is unidimensional, Theorem 1 simplies to:

Corollary 1: Suppose $X \subset E^1$. Then the indirect profile on Π is single-peaked with respect to $\leq_o$ for every profile in $R(n)$ if and only if each voter's predictive map is either

1) constant, or

2) strictly monotonic.

Theorem 1 and Corollary 1 contain both possibility and impossibility results in their "if" and "only if" statements, respectively. These results specify exact limits for spatial voting models that posit a linearly ordered set of predictive elements and economic decision makers. These limits are natural for certain contexts (e.g., the agency budget problems in Hinich and Mackay [1979]). Therefore, in the next section, we will take advantage of this theorm by obtaining the median voter results which follow within these limits.

3. Median Voter Results

We say that a candidate is "identified with" a particular $\pi \in \Pi$ whenever this element of the predictive set is the one which (all voters agree) specifies the candidate's location in Π. In addition, we say that a candidate θ identified with $\pi_\theta \in \Pi$ receives at least as many votes as a candidate ψ identified with $\pi_\psi \in \Pi$ if and only if

$$(3.1) \qquad \#\{i \in N \mid (\pi_\theta, \pi_\psi) \in P(Q_i)\} \geq \#\{i \in N \mid (\pi_\psi, \pi_\theta) \in P(Q_i)\} \quad .$$

This is appropriate whenever every voter is one who casts a ballot if and only if he strictly prefers one of the candidates to the other (and that he votes for the one he prefers). It is also appropriate (in terms of expected votes) when, alternatively, indifferent individuals vote but there is a probability of 1/2 of his voting for each candidate (and individuals with strict preferences vote for their preferred alternative).

To apply results due to Denzau and Parks [1975] we now only have to let $c_1, \ldots, c_{2k}$ be a rearrangement of the sequence $a_1, b_1, a_2, b_2, \ldots, a_k, b_k$ such that $c_1 \leq c_2 \leq \cdots \leq c_{2k}$. Then

<u>Theorem 2</u>: Suppose that $(R_1,\ldots,R_n) \in R(n)$ and every voter's predictive map either 1) is constant, or 2) has its range contained in a linear interval and is strictly monotonic on this interval. Then a candidate θ identified with $\pi_\theta \in \Pi$ receives at least as many votes as any other candidate ψ identified with a $\pi_\psi \in \Pi$ if and only if $c_k \leq \pi_\theta \leq c_{k+1}$.

As in Section 2, when the society's policy space is unidimensional this result simplifies. In this case we have:

Corollary 2: Suppose that $X \subset E^1$, $(R_1,\ldots,R_n) \in R(n)$, and every voter's predictive map is either constant or strictly monotonic. Then a candidate θ identified with $\pi_\theta \in \Pi$ receives at least as many votes as any other candidate ψ identified with a $\pi_\psi \in \Pi$ if and only if $c_k \leq \pi_\theta \leq c_{k+1}$.

In either case, there is (necessarily) some π_θ which satisfies the given inequality. Furthermore, any such π_θ is, by its very definition, a median for the distribution of c_j's. Therefore, Theorem 2 and Corollary 2 provide both existence and location results for "non-losing identifications" in the set of predictive elements.

4. Conclusion

The analysis provided in this paper has determined conditions on the voters' predictive maps which are both necessary and sufficient to assure that their indirect preferences on the predictive set are always single-peaked (with respect to the natural linear order for this set) in such societies. In other words, this paper has obtained exact properties which the voters' predictive maps must satisfy for the implications of single-peakedness (with respect to the natural linear order for a set) to be generally applied to the societies we're studying

(i.e., without making any further "domain restrictions" on the preference patterns in these societies). This has led to certain new median voter results. Furthermore, whenever a society has a predictive set which is a closed interval in the real line, the results of Hinich, Mackay and Pollard described in the introduction follow as corollaries of these results.

Finally, while the analysis in this paper has successfully extended the existing results on spatial voting models with predictive elements, it has also revealed some important limitations which can be involved whenever an analysis starts out with a predictive dimension and single peaked preferences along this dimension (such as the analyses with an ideological continuum in Downs [1957], McKenzie and Tullock [1978] and Mueller [1979], for instance).

APPENDIX

Proof of Theorem 1:

("if"): We will proceed by showing that the indirect preferences for each $i \in N$ must satisfy the conditions given by (2.2).

Suppose $w_i(\pi)$ is constant for a particular $i \in N$. Then i is indifferent over all of the alternatives in Π. Therefore, since Π is compact, the minimal and maximal elements of Π (with respect to $\leq_o$) will serve as an a_i and as a b_i, respectively, for the peak of his preference relation (see (2.2)). Because of the antisymmetry of $\leq_o$, these elements are unique.

Alternatively, consider any $i \in N$ whose predictive map, $w_i(\pi)$, is such that its range is contained in a linear interval and is strictly monotonic on this interval. Transitivity, reflexivity and connectedness follow immediately for Q_i. We will show that Q_i also inherits regularity and certain other properties (which are extensions for linearly ordered sets of the concepts of convexity and weak convexity, respectively). First, regularity: For any $\pi_1 \in \Pi$, the upper contour set $Q_i(\pi_1)$ is defined by

$$Q_i(\pi_1) = \{\pi \in \Pi: (\pi,\pi_1) \in Q_i\} \quad .$$

Therefore,

$$Q_i(\pi_1) = \{\pi \in \Pi: (w_i(\pi),w_i(\pi_1)) \in R_i\} \quad .$$

Let $C(w_i)$ be the range (or co-domain) of w_i. Then, using the notation

$$w_i^{-1}(A) = \{\pi \in \Pi: w_i(\pi) \in A\} \quad (\text{for } A \subseteq C(w_i)) ,$$

we have

$$Q_i(\pi_1) = w_i^{-1}(\{x \in C(w_i): (x, w_i(\pi_1)) \in R_i\}) .$$

But, since R_i is regular, the upper contour set

$$R_i(w_i(\pi_1)) = \{x \in X: (x, w_i(\pi_1)) \in R_i\}$$

is closed. Furthermore, since $w_i(\pi)$ is continuous and Π is compact, $C(w_i)$ is compact. Hence,

$$\{x \in C(w_i): (x, w_i(\pi_1)) \in R_i\} = R_i(w_i(\pi_1)) \cap C(w_i)$$

is closed. Therefore, since $w_i(\pi)$ is continuous, $Q_i(\pi_1)$ is closed. Thus Q_i is regular.

Next we will show that, for any $\pi_1, \pi_2, \pi_3 \in \Pi$,

(A.1) $\quad [(\pi_3, \pi_1) \in P(Q_i) \text{ and } \pi_1 <_o \pi_2 <_o \pi_3] \Rightarrow [(\pi_2, \pi_1) \in P(Q_i)]$.

Suppose that $(\pi_3, \pi_1) \in P(Q_i)$ and $\pi_1 <_o \pi_2 <_o \pi_3$ for some $\pi_1, \pi_2, \pi_3 \in \Pi$.

Then $(w_i(\pi_3),w_i(\pi_1)) \in P(R_i)$. Also, since $w_i(\pi)$ has its range contained in a closed linear interval and is strictly monotonic on that interval, there must be some $\alpha \in (0,1)$ such that

$$w_i(\pi_2) = w_i(\pi_1) + (1 - \alpha)w_i(\pi_3) \quad .$$

But, by the convexity of R_i ,

$$(w_i(\pi_3),\ w_i(\pi_1)) \in P(R_i)$$

implies

$$\left(\alpha w_i(\pi_1) + (1 - \alpha)w_i(\pi_3),w_i(\pi_1)\right) \in P(R_i) \quad .$$

Therefore,

$$(w_i(\pi_2),\ w_i(\pi_1)) \in P(R_i) \quad .$$

Hence,

$$(\pi_2,\pi_1) \in P(Q_i) \quad .$$

Finally, a similar argument (using the weak convexity of R_i) establishes that, for any π_1, π_2, $\pi_3 \in \Pi$,

(A.2) $$[(\pi_3,\pi_1) \in Q_i \text{ and } \pi_1 <_o \pi_2 <_o \pi_3] \Rightarrow [(\pi_2,\pi_1) \in Q_i] \quad .$$

We will now show that these properties imply that i's indirect preferences on Π satisfy the conditions which each individual's preferences must satisfy for single-peakedness with respect to the order $\leq_o$ to hold. First the inherited properties of Q_i together with the compactness of Π imply that there is at least one maximal element in Π (with respect to Q_i), i.e.,

$$M_i = \left\{\pi \in \Pi\colon (\pi,\pi_1) \in Q_i,\ \forall\, \pi_1 \in \Pi\right\}$$

is non-empty (e.g., see Bergstrom [1975; Lemma, p. 403]). The inherited properties also imply that

$$M_i = \left\{\pi \in \Pi\colon (\pi,\pi_1) \in Q_i\right\} \qquad \text{for any } \pi_1 \in M_i \ .$$

Therefore, by the regularity of Q_i, M_i is closed. Hence, since Π is compact, M_i is also compact. Therefore, since (A.2) also holds,[14/] there exists a_i, $b_i \in \Pi$ with $a_i \leq_o b_i$ such that

$$M_i = \left\{\pi \in \Pi\colon a_i \leq_o \pi \leq_o b_i\right\} \ .$$

Since $\leq_o$ is anti-symmetric, these "endpoints" are unique.

We now turn to the specific conditions given in (2.2).

a) Suppose $\pi_1 <_o \pi_2 \leq_o a_i$. Then $\pi_1 \notin M_i$. Therefore, $(\pi_1,a_i) \notin Q_i$. Therefore, by connectedness, $(a_i,\pi_1) \in P(Q_i)$. Consequently, if $\pi_2 = a_i$ we have $(\pi_2,\pi_1) \in P(Q_i)$ immediately. Alternatively, if $\pi_2 \neq a_i$, then $\pi_1 <_o \pi_2 <_o a_i$. Therefore, by (A.1), $(\pi_2,\pi_1) \in P(Q_i)$.

b) Suppose $a_i \leq_o \pi_1 \leq_o \pi_2 \leq_o b_i$. Then $\pi_1, \pi_2 \in M_i$. Therefore, $(\pi_1,\pi_2) \in Q_i$ and $(\pi_2,\pi_1) \in Q_i$. Hence, $(\pi_1,\pi_2) \in I(Q_i)$.

c) Similar to a).

d) Suppose $\pi_1 <_o a_i$ and $a_i \leq_o \pi_2 \leq_o b_i$. Then $\pi_2 \in M_i$ and $\pi_1 \notin M_i$. Therefore, $(\pi_1,\pi_2) \notin Q_i$. Hence, by the connectedness of Q_i, $(\pi_2,\pi_1) \in P(Q_i)$

A similar argument holds for $\pi_1 <_o b_i$ and $a_i \leq_o \pi_2 \leq_o b_i$.

("only if"):

We will proceed by considering a sequence of possible assumptions about the voters' predictive maps. We will show that each of these assumptions, in turn, allows the theorem's single-peakedness requirement to be violated at some profile in $R(n)$. In particular, since single-peakedness is a condition which requires certain properties to be satisfied by <u>each</u> voter's preferences, we will be able to show this in each case by showing there is at least one profile such that at least one voter's indirect preferences will violate the conditions in (2.2). This successive elimination of alternative assumptions will establish our result.

We will begin by showing that the range of each $w_i(\pi)$ must be contained in a linear interval. Consider any three elements in Π such that $\pi_1 <_o \pi_2 <_o \pi_3$. We will show, more specifically, that $w_i(\pi_2)$ must be a linear combination of $w_i(\pi_1)$ and $w_i(\pi_3)$ lie on a common indifference curve while $w_i(\pi_2)$ is in the strict lower contour set for this curve (for examples with $X \subset E^2$, see Figure 4). But then

$$(\pi_1,\pi_2) \in P(Q_i)$$

and

$$(\pi_3,\pi_2) \in P(Q_i) \quad ,$$

which implies that single-peakedness cannot be satisfied by the corresponding indirect profiles on Π. Our claim now follows since, as noted before, the range of $w_i(\pi)$ must be compact.

Finally, suppose that, for some $i \in N$, $w_i(\pi)$ is neither constant nor strictly monotonic. We consider first whether $w_i(\pi)$ can be constant on any strict subset of Π which is connected (with respect to the order topology on Π) and has at least two elements. Suppose there is such a set, A. Let $\pi_1, \pi_2 \in A$, with $\pi_1 <_o \pi_2$. Then there will also be some profile in each $R(n)$ for which i has

(1) $(x,w_i(\pi_1)) \in P(R_i)$ for some $x = w_i(\pi_0)$ with $\pi_0 \in \Pi - A$ and $\pi_0 >_o \pi_1$, or

(2) $(x,w_i(\pi_2)) \in P(R_i)$ for some $x = w_i(\pi_3)$ with $\pi_3 \in \Pi - A$ and $\pi_3 >_o \pi_2$

(since A is a strict subset of Π). Finally, suppose that the indirect profiles on Π is single-peaked. Then we must have, for these cases, (1) $b_i <_o \pi_1 <_o \pi_2$ or $\pi_1 <_o \pi_2 <_o a_i$ (see (2)). But $w_i(\pi_1) = w_i(\pi_2)$, so we must have $(\Pi_1, \Pi_2) \in I(Q_i)$ as well, a contradiction. For an illustration, see Figure 5 vs. Figure 1.

Therefore, for $w_i(\pi)$ to not be strictly monotonic, there must exist three elements

$$\pi_1, \pi_2, \pi_3 \in \Pi \quad \text{with} \quad \pi_1 <_o \pi_2 <_o \pi_3$$

and either

$$(1) \qquad w_i(\pi_1) >_\alpha w_i(\pi_2) \quad \text{and} \quad w_i(\pi_3) >_\alpha w_i(\pi_2) \quad ,$$

or

$$(2) \qquad w_i(\pi_2) >_\alpha w_i(\pi_1) \quad \text{and} \quad w_i(\pi_2) >_\alpha w_i(\pi_3) \quad .$$

Suppose the first holds. Then there also exists some profile in each $R(n)$ such that, for all $x, y \in C(w_i)$, $(x,y) \in R_i$ if and only if $x \geq_\alpha y$. But then

$$(\pi_1, \pi_2) \in P(Q_i) \quad \text{and} \quad (\pi_3, \pi_2) \in P(Q_i)$$

and the indirect profile on Π can't be single-peaked along $\leq_o$.

Alternatively, suppose the second holds. Then there also exists some profile in $Q(n)$ such that, for all $x, y \in C(w_i)$, $(x,y) \in R_i$ if and only if $y \geq_\alpha x$. But then

$$(\pi_1, \pi_2) \in P(Q_i) \quad \text{and} \quad (\pi_3, \pi_2) \in P(Q_i)$$

again. For an illustration with $X \subset E^2$, see Figure 6.

Therefore $w_i(\pi)$ is either constant or strictly monotonic.

Q.E.D.

Proof of Theorem 2:

Follows from Theorem 1 (above) and Theorem 2 in Denzau and Parks [1975].

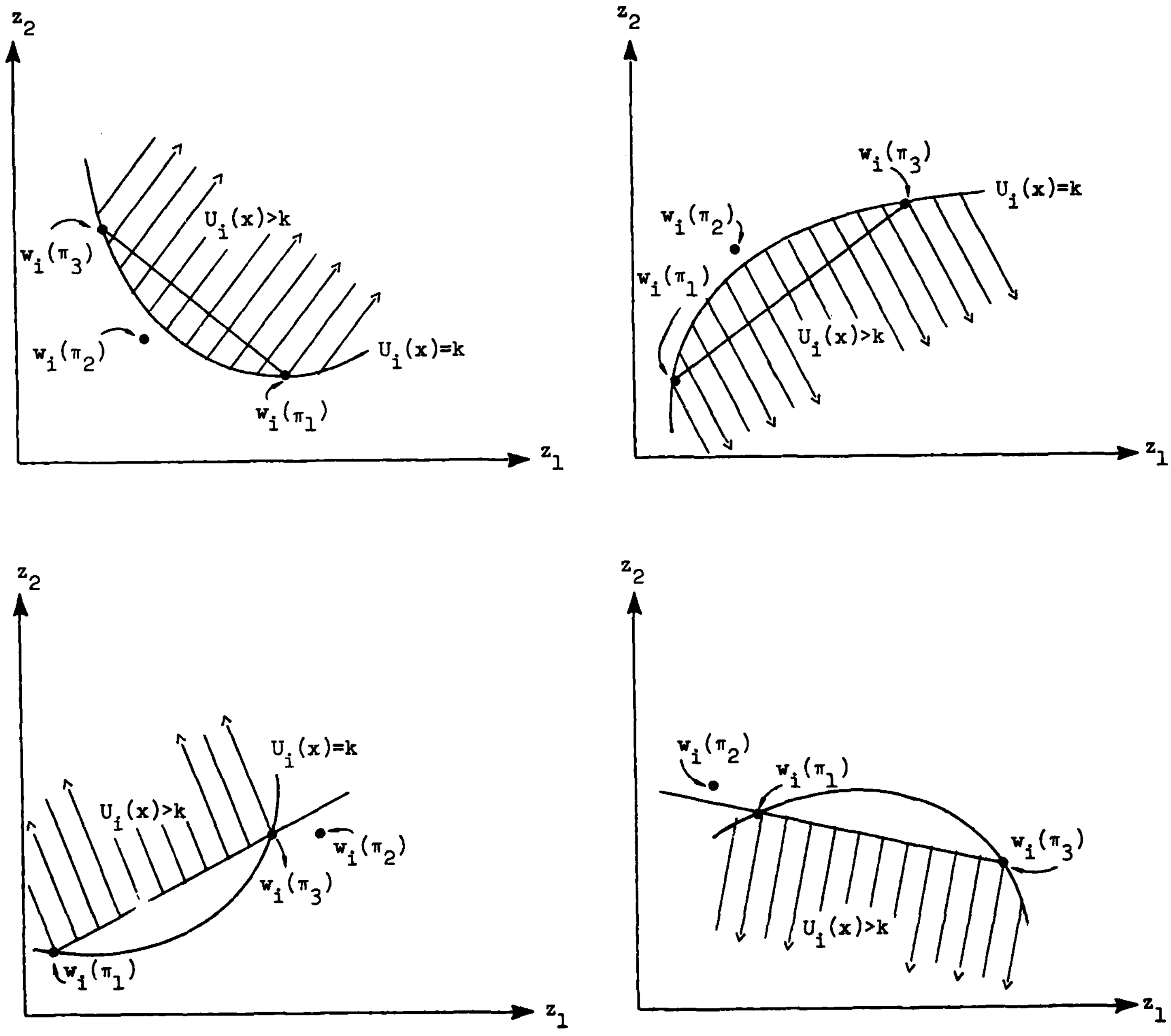


Figure 4

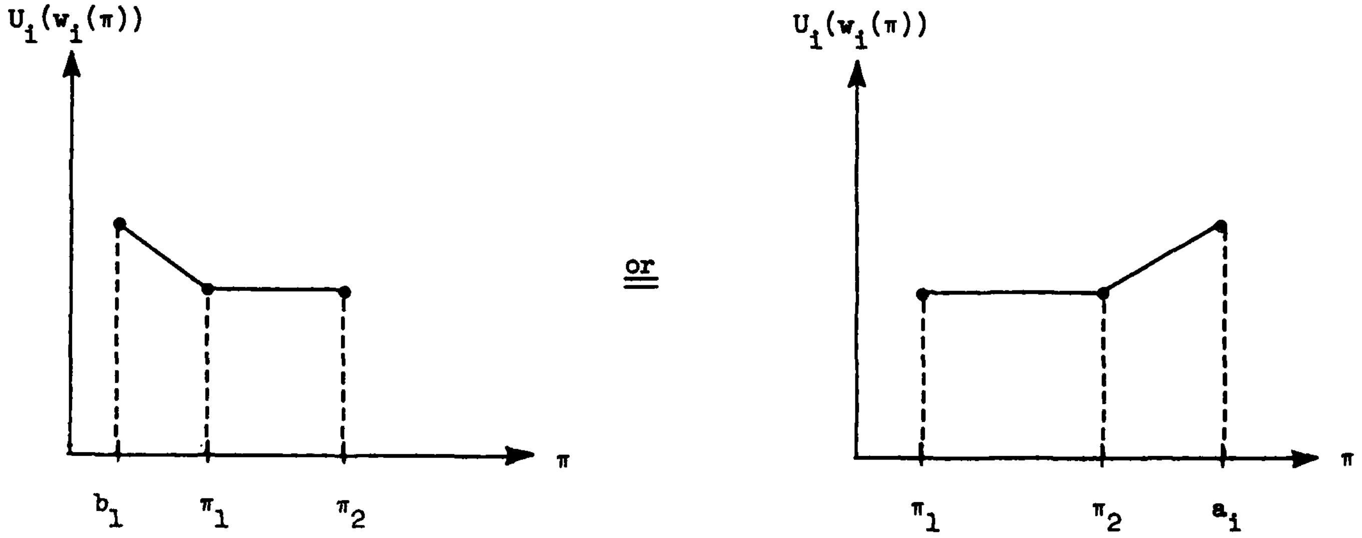
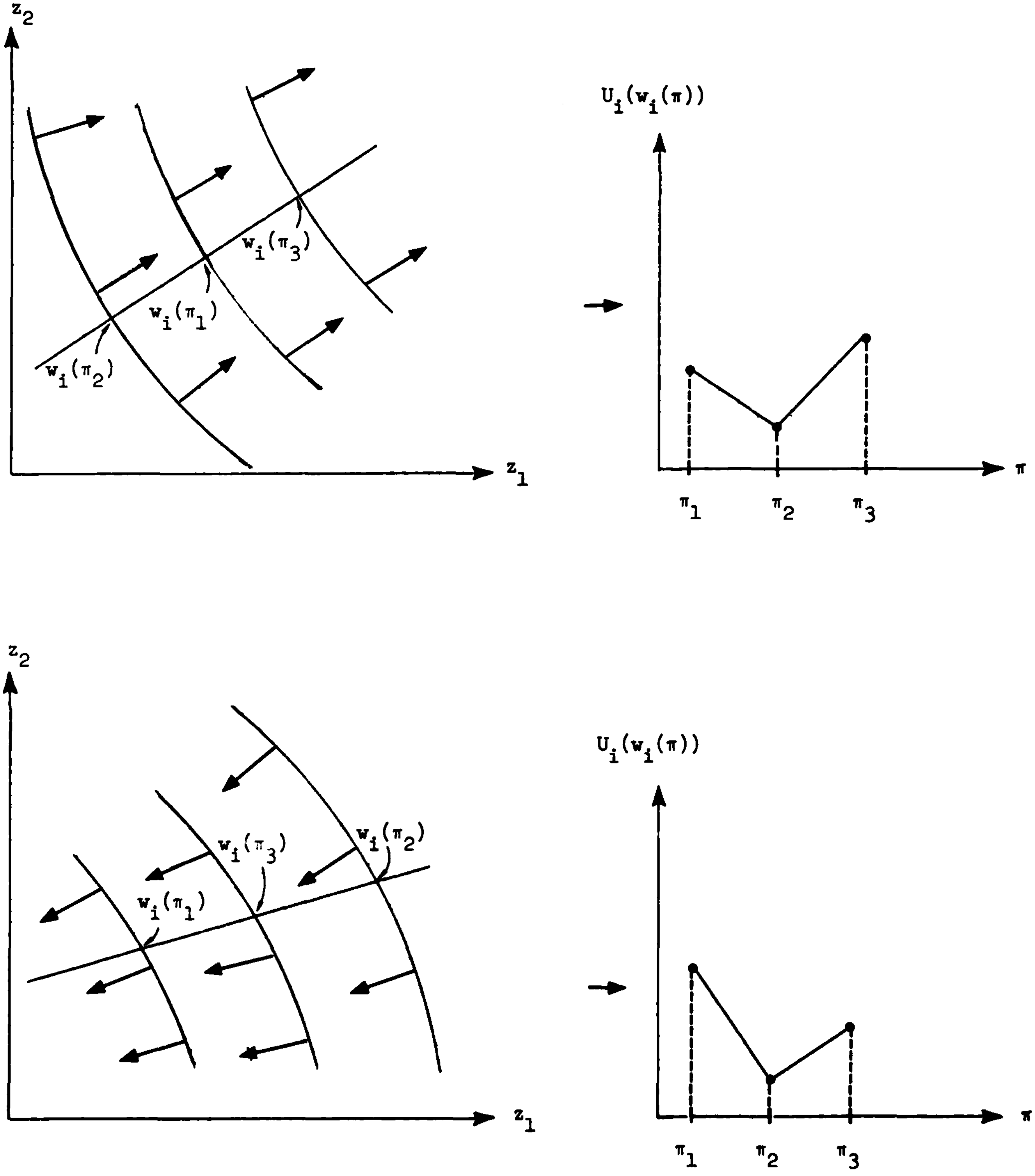


Figure 5



z_2
z_1
$w_i(\pi_3)$
$w_i(\pi_1)$
$w_i(\pi_2)$
$U_i(w_i(\pi))$
π
π_1
π_2
π_3
z_2
z_1
$w_i(\pi_3)$
$w_i(\pi_2)$
$w_i(\pi_1)$
$U_i(w_i(\pi))$
π
π_1
π_2
π_3

Figure 6

Footnotes

1/ For surveys of some of this work, the reader is referred to Davis, Hinish and Ordeshook [1970], Plott [1971], Mueller [1976, 1979], Kramer [1977] and Feldman [1980].

2/ Quoting from Downs [1957] (p. 99): "A citizen who regards ideologies as cost-saving devices is not employing them as a last resort. They are to him a first resort, used to save the cost of calculating his party differential" (i.e., the differences between candidates on the issues). "Since it is much cheaper to keep informed about ideologies than about issues ... he does the former as a rational shortcut to the second."

3/ Let $X \subset R^m$ be the society's economic policy space. Let $R_i \subset X \times X$ be the preferences of some individual i on this space. Then to say that i has "ellipsoidal preferences" on X means that there exists some symmetric, positive-definite, $(m \times m)$ matrix A_i and a policy $x_i \in X$ such that

$$(x,y) \in R_i \iff (x - x_i)'A_i(x - x_i) \leq (y - x_i)'A_i(y - x_i)$$

for any $x, y \in X$. x_i is called an "ideal point" for individual i. Such preferences have previously been studied in Davis and Hinich [1966, 1968], Davis, DeGroot and Hinich [1972], and elsewhere.

4/ For an exact definition of this phrase, see Section 3.

5/ Let $R \subset X \times X$ be a preference relation on X. Then R is

(i) "reflexive" if and only if $(x,x) \in R, \forall\ x, y \in X$;

(ii) "connected" if and only if $(x,y) \in R$ or $(y,x) \in R, \forall\ x, y \in X$;

(iii) "transitive" if and only if $(x,y) \in R$ and $(y,z) \in R$ imply $(x,z) \in R, \forall\ x, y, z \in X$;

(iv) "regular" if and only if $\{y \in X: (y,x) \in R\}$ is closed, $\forall\ x \in X$;

(v) "weakly convex" if and only if $(x,y) \in R$ implies $(\lambda x + (1 - \lambda)y,y) \in R, \forall\ x, y \in X$ and $\lambda \in (0,1)$;

(vi) "convex" if and only if $(x,y) \in P(R)$, implies $(\lambda x + (1 - \lambda)y,y) \in P(R), \forall\ x, y \in X$ and $\lambda \in (0,1)$ (see footnote 5).

6/ I.e., $(x,y) \in P(R_i)$ if and only if $(x,y) \in R_i$ and $(y,x) \notin R_i$.

7/ I.e., $(x,y) \in I(R_i)$ if and only if $(x,y) \in R_i$ and $(y,x) \in R_i$.

8/ A binary relation $\leq_o$ on Π is a "linear order" if and only if $\leq_o$ is transitive, connected and antisymmetric (i.e., $\pi_1 \leq_o \pi_2$ and $\pi_2 \leq_o \pi_1$ implies $\pi_1 = \pi_2$ for any $\pi_1, \pi_2 \in \Pi$).

9/ The "order topology" on a linearly ordered set Π is defined by the subbase of open sets $\{\pi \in \Pi: \pi < \pi_1\}$, $\{\pi \in \Pi: \pi > \pi_1\}$, $\pi_1 \in \Pi$.

10/ It should be observed that, when Π is a finite set, the continuity assumption on $w(\pi)$ doesn't impose any restriction at all. In this case, each singleton set in Π (and, hence, every subset of Π) is both open and closed.

11/ This definition is based on the earlier definitions due to Black [1958], Arrow [1963] and Fishburn [1973]. It differs from the Arrow definition since there can be more than two maximal elements when Π is finite and more than one maximal element when Π is convex. For a careful discussion of how it differs from Fishburn's definition, see Denzau and Parks [1975].

12/ Of course, these linear intervals may vary from voter to voter.

13/ I.e., there is no strictly weaker condition on the predictive maps which will be sufficient for single-peakedness with respect to $\leq_o$ to occur for every pair in $R(n) \times \overline{W}(n)$.

14/ To illustrate the role of (7), we include the following examples:

(i) $\Pi = \{0, 1/2, 1\}$, $0 <_o 1/2 <_o 1$, and $U_i(0) = 1$, $U_i(1/2) = 0$, $U_i(1) = 1$ for some $i \in N$,

(ii) $\Pi = [0,1]$, $\leq_o$ is the linear ordering given by the numerical values of the $\pi \in \Pi$, and $U_i(\pi) = \begin{cases} \pi & \text{for } 0 < \pi \leq 1 \\ 1 & \text{for } \pi = 0 \end{cases}$.

In both examples, the individual's preferences satisfy (7), but not (8). They also satisfy reflexivity, connectedness, transitivity and regularity. But, in both cases the set of maximal elements for i is $\{0,1\}$. Therefore preferences are not single-peaked along the given linear orders for Π.

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